Exploring Transformations

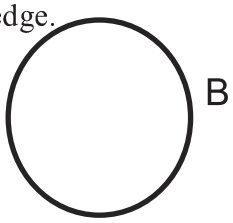
Materials: ruler, Mira, protractor, paper, pencil

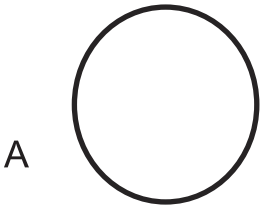
Activity 1: REFLECTIONS

A Mira is a plastic device that acts like a mirror that you can see through. A Mira reflects objects, just as a mirror does, but since the Mira is transparent, the image of an object reflected in it also appears behind the Mira.

The drawing edge of a Mira is beveled. When using a Mira, it should be placed with the beveled edge down. Look directly through the Mira from the side with the beveled edge to locate the image of the object behind the Mira.

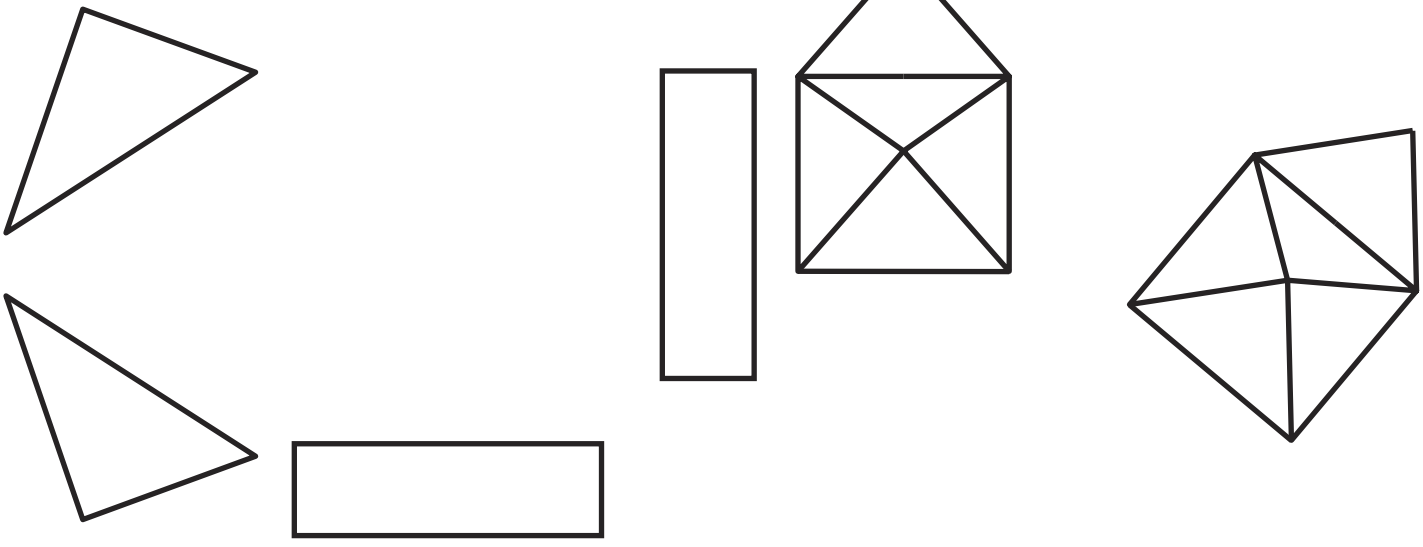
1. Place your Mira so that the image of circle A fits on circle B. Hold the Mira steady with one hand, and draw a line along the drawing edge.



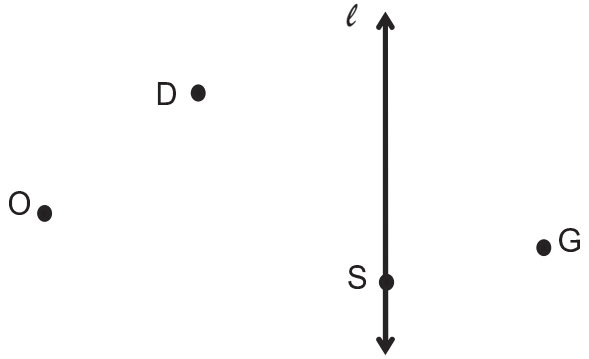


Take away the Mira. The line you have drawn is called the *Mira line*. It represents the line of reflection.

2. For each pair of figures below, fit the image of one of the figures onto the other, and draw the Mira line.



3. Use a Mira to find the reflection of each point – D, O, G, and S – through line ℓ (the Mira should be on line ℓ) Use the prime notation to name each image point. For example, the image of point D would be named D'.



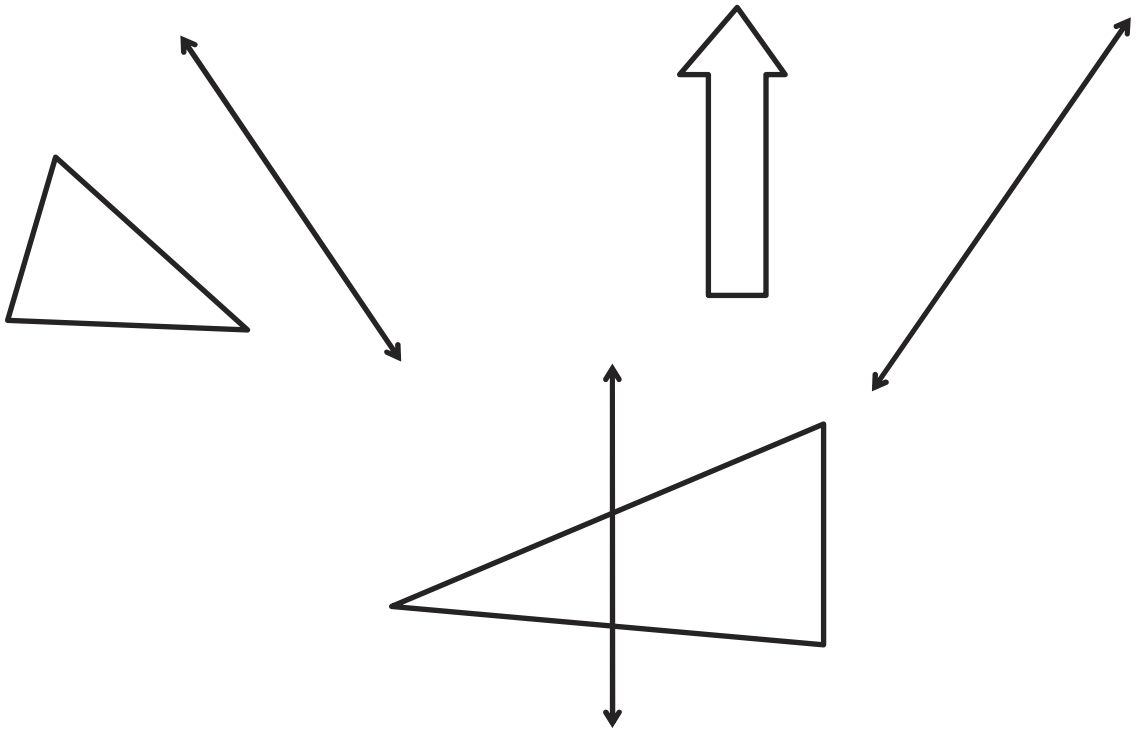
Draw line segments $\overline{DD'}$, $\overline{OO'}$, and $\overline{GG'}$. Label the points where line ℓ intersects these segments C, A, and T, respectively.

a. What is the relationship between line ℓ and $\overline{DD'}$, $\overline{OO'}$, and $\overline{GG'}$?

b. Where is point S located in relation to line ℓ?

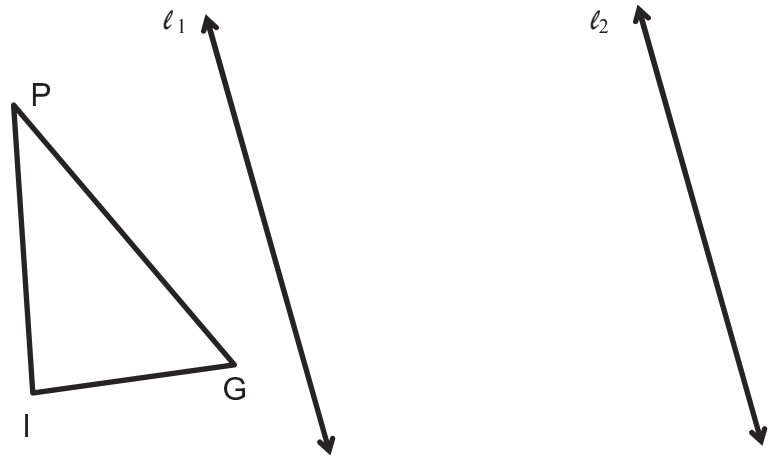
c. What is the relationship between points S and S'?

4. Use a Mira to draw the reflection of each figure through the given line.



Activity 2: TRANSLATIONS

1. a. Reflect ΔPIG through line ℓ_1. Let C, O, and W denote the images of P, I, and G, respectively.



b. Reflect ΔCOW through line ℓ_2. Let H, E, and N denote the images of C, O, and W, respectively.

c. Draw line segments $\overline{PH}$, $\overline{IE}$, and $\overline{GN}$.

d. Make a tracing of Δ PIG, and slide it onto ΔHEN by moving its vertices along the three "tracks" you have just drawn. Is it necessary to flip or turn the tracing to do this? _____

e. What two relationships do the "tracks" seem to have?

The transformation in the previous problem is called a *translation*. The problem illustrates the following definition:

A translation is the composite of two reflections through parallel lines.

2. Which of the following are preserved by a translation? by a reflection?

a. collinearity of points

b. betweenness of points

c. angle measurement

d. distance between points

e. orientation of a figure

f. congruence of figures

Activity 3: ROTATIONS

You have studied two basic geometric transformations - reflections and translations. The drawings in Figure A illustrate a third transformation. The two figures are identical, but the duck is neither a reflection nor a translation of the rabbit.

To illustrate the relationship between the rabbit and the duck, place a piece of paper over the rabbit, and pin it at point P. Trace over the rabbit, then turn the paper about the pin until the rabbit coincides with the duck. This should explain why the duck is called a *rotation image* of the rabbit.

Figure A

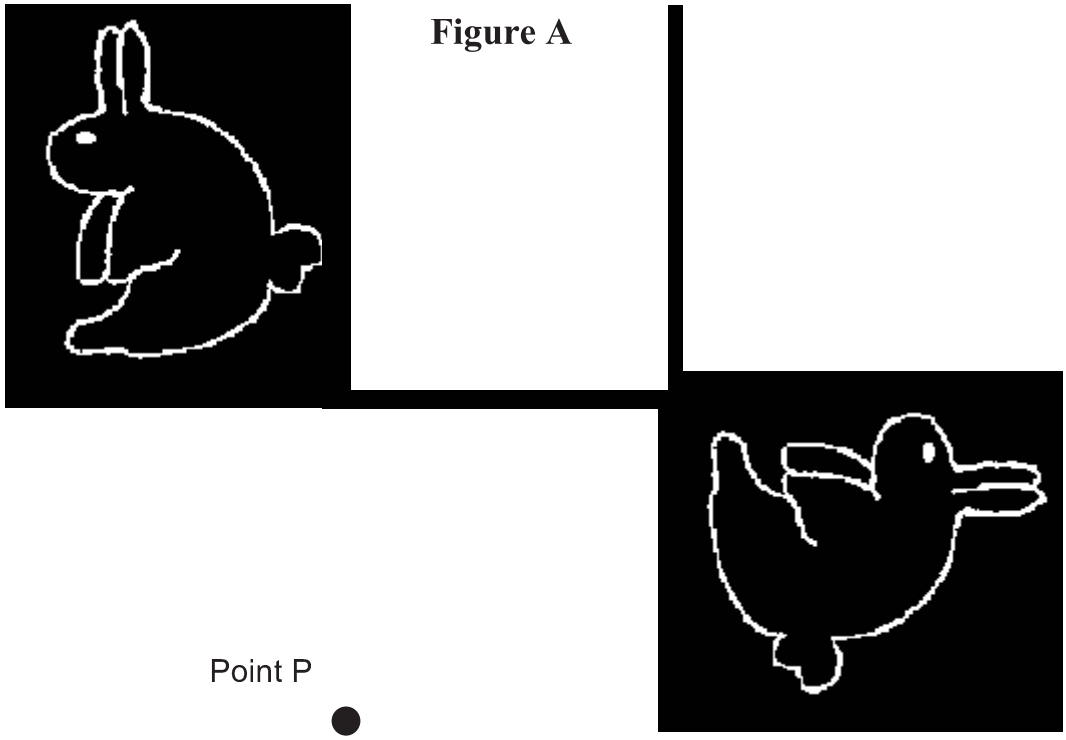
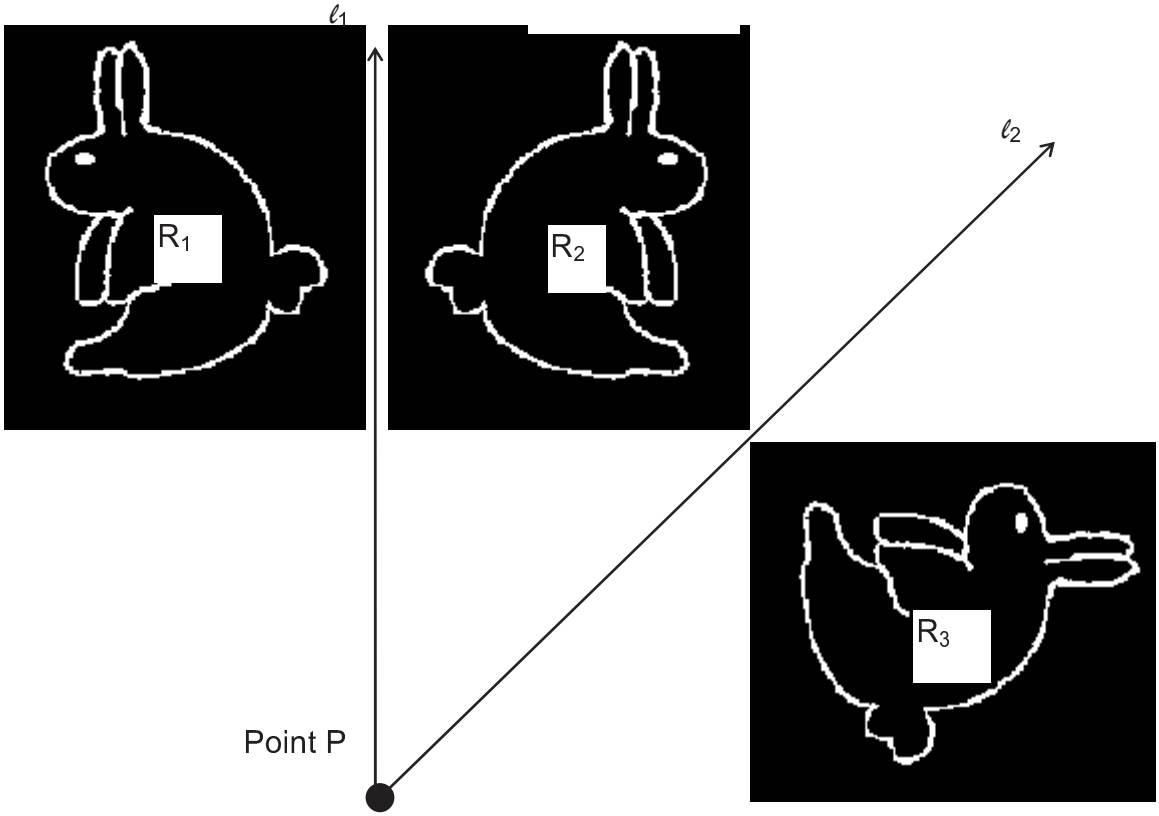


Figure B

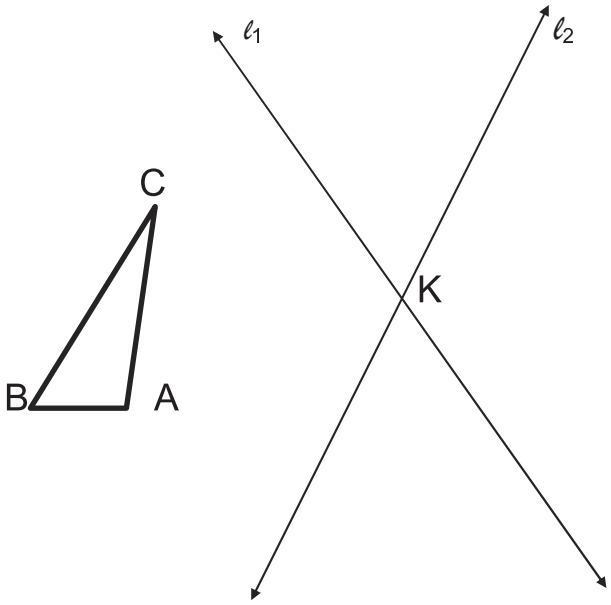
Reflections can also be used to illustrate the relationship between the rabbit and the duck. The drawings in Figure B show that, if the rabbit, R_1, is reflected through line ℓ_1, and if its image R_2, is reflected through line ℓ_2, then the result is the duck, R_3. Verify this with your Mira.



As the preceding example shows, the rotation that transforms the rabbit into the duck is the composite of two reflections through intersecting lines. This illustrates the following definitions:

A rotation is the composite of two reflections through intersecting lines.
The center of rotation is the point at which the two lines intersect.

1. What is the measure of the acute angle formed by lines ℓ_1 and ℓ_2 below?



2. Reflect ΔABC by through line ℓ_1. Label the reflection ΔDEF and then reflecting ΔDEF through line ℓ_2. Let ΔXYZ be the reflection of ΔDEF. ΔXYZ will be the **rotation** of ΔABC through lines ℓ_1 and ℓ_2.

   a. Draw and measure $\angle BKY$.
   Note that the vertex of $\angle BKY$ is at the center of rotation and that one side of the angle contains point B on the original figure and the other side contains the rotation image, Y, of the point B. The measure of an angle formed this way is called the ***magnitude*** of the rotation.

   b. how does the magnitude of rotation compare with your answer to problem 1?

   c. In what direction, clockwise or counterclockwise, was ΔABC rotated?

3. Now rotate ΔABC by reflecting it through line ℓ_2 and then reflecting the reflection through line ℓ_1. Let R, S, and T be the rotation images of points A, B, and C, respectively.

   a. What is the magnitude of the rotation? How does it compare to problem 1?

   b. In what direction was ΔABC rotated?

4. Which of the following are preserved by a rotation?

   a. collinearity of points

   b. betweenness of points

   c. angle measure

   d. distance between points

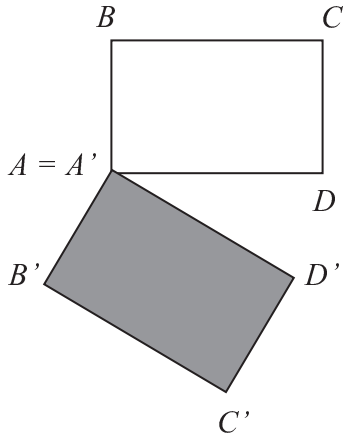
   e. orientation of a figure

   f. congruence of figures

Activity 4: IDENTIFYING TRANSFORMATIONS

Identify which transformation (reflection, translation, or rotation) or combination of transformations, if possible, that would change each white figure shown to the corresponding gray image.

1.



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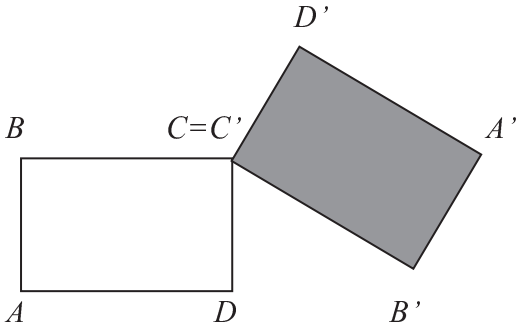

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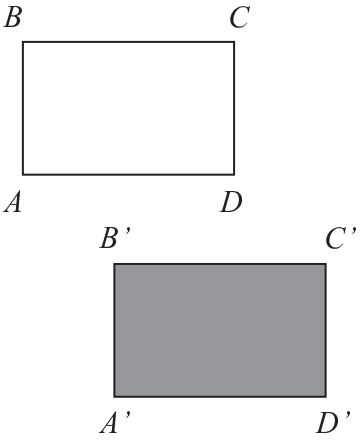

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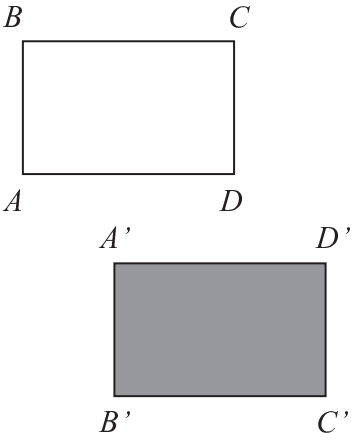

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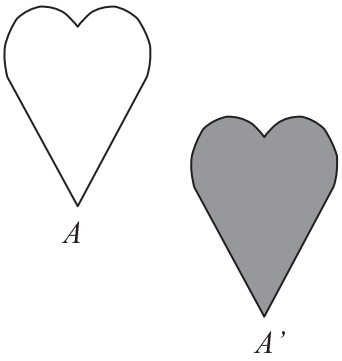

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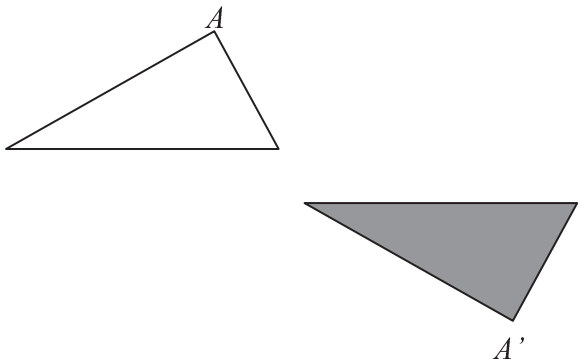

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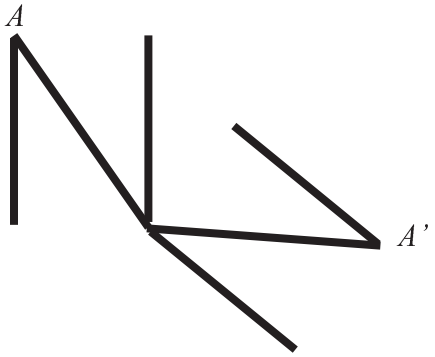

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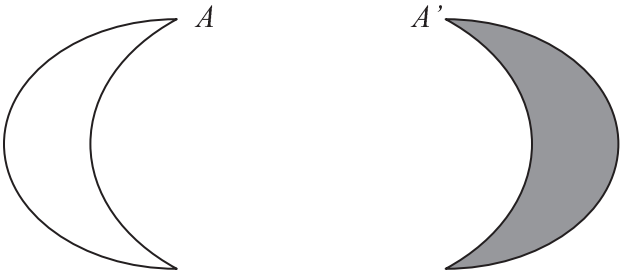

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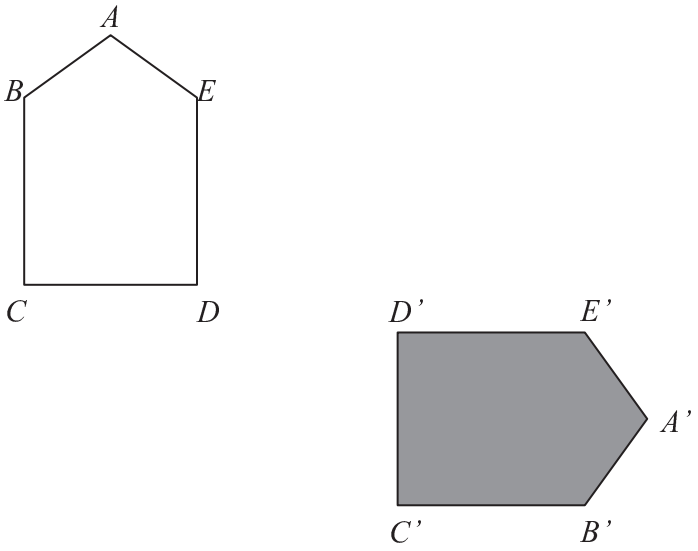

10.